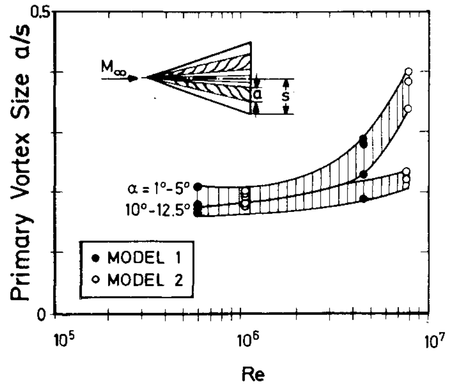

Fig. 8 Influence of *Re* on vortex size, $M_\infty = 3.5$.

position and pressure level has been observed. This is certainly true for the flow type with leading-edge separation and agrees in general with results of Ref. 8. However, for the type of flow described as separation with shock, further considerations are worthwhile. The experiments seem to show that the separation bubble flattens for higher Reynolds numbers, moving closer to the upper surface until the flow near the leading edge is attached. Thus another type of flow, the well-known shock-induced separation, is established. According to the $\alpha_N - M_N$ diagram of Fig. 1 and the preceding conclusions, shock-induced separation is obtained at either higher Mach numbers or higher Reynolds numbers. The $\alpha_N - M_N$ diagram is therefore strictly valid only for fixed Reynolds numbers, a fact which is also confirmed by results at hypersonic Mach numbers.

Conclusion

The influence of Reynolds number on the types of flow left and right to the Stanbrook-Squire boundary has been examined. For leading-edge separation conditions, the vortex position and intensity, and thus the suction pressure, vary while the type of flow remains nearly unchanged. In the region of separation with embedded shock, *Re* affects not only the shape of the separation bubble and pressure level near the leading edge but the type of flow as well. At sufficiently high Reynolds numbers the flow type of separation with shock changes to one with shock-induced separation. In agreement with hypersonic investigations it turns out that the flow regimes sketched in the $\alpha_N - M_N$ diagram are effective only for fixed *Re*.

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Real Gas Effects in a Pulsed Laser Propulsion System

Sharad Chandra Purohit*
Vikram Sarabhai Space Centre,
Trivandrum, India

RECENTLY, Simons and Pirri[1] developed a fluid mechanical model for a pulsed laser propulsion system and showed that it is an efficient steady-state system. One of the assumptions made by them is that the gas is ideal. The purpose of this Note is to study the change in the functional values of energy efficiency and specific impulse when the gas is almost ideal.

The modified equation of state in a simplified form, as given by Landau and Lifshitz,[2] is

$$p = \rho RT(1 + b\rho) \tag{1}$$

where b is the molecular weight. Using notation similar to that of Simons and Pirri, the basic transformed equations can be written as

$$\frac{dg}{d\eta} - \frac{hf}{2\beta} + \frac{f}{\beta}[(1-\beta)h - \eta]\frac{dh}{d\eta} = 0 \tag{2}$$

$$\frac{dh}{d\eta} - \frac{2}{(1-\beta)} - \frac{\eta}{(1-\beta)}\frac{1}{f}\frac{df}{d\eta} + \frac{h}{f}\frac{df}{d\eta} + 2\frac{h}{\eta} = 0 \tag{3}$$

$$\begin{aligned}[(1-\beta)h - \eta)]\frac{dg}{d\eta} - g\frac{df}{d\eta}[(1-\beta)h - \eta]\Big[\frac{\alpha}{\beta} + \frac{\gamma}{f} \\ + \frac{\alpha}{\beta(\gamma-1)}\Big(1 + \frac{\alpha}{\beta(\gamma-1)}f\Big)^{-1}\Big] + g\Big[2\gamma - 3 + 2f\frac{\alpha}{\beta} \\ + 2f\frac{\alpha}{\beta(\gamma-1)}\Big(1 + \frac{\alpha}{\beta(\gamma-1)}f\Big)^{-1}\Big] = 0\end{aligned} \tag{4}$$

While strong shock conditions are defined as (see Rao and Purohit[3]):

$$\begin{aligned}\rho_s(t) &= \rho_\ell(t)/\beta \\ p_s(t) &= (1-\beta)\rho_\ell(t)V_s^2(t) \\ u_s(t) &= (1-\beta)\ V_s(t)\end{aligned} \tag{5}$$

where

$$\beta = \frac{(\gamma - 1 - \alpha) + \sqrt{(\gamma - 1 - \alpha)^2 + 4\ \alpha\ (\gamma + 1)}}{2(\gamma + 1)}$$

$$\alpha = b\ \rho_\ell(t)\ \ (\gamma - 1)$$

α is a nonideal gas parameter and $\alpha = 0$ is a particular case discussed by Simons and Pirri. β is a monotonically increasing function of positive α.

By using a fourth-order Runge-Kutta procedure, the basic equations (2-4) are solved from $\eta = 1$ to $\eta = 0$ together with the

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*Scientist, Applied Mathematics Section.

Table 1 Values of energy efficiency, e_R

α/γ	1.1	1.25	1.40	1.67
0.0	0.770774	0.981932	0.996722	0.959639
0.00005	0.764761	0.981826	0.996853	0.961653
0.0001	0.760494	0.980106	0.998962	0.963174
0.0005	0.612748	0.968819	1.007108	0.977321
0.001	0.341371	0.929104	1.025500	0.997332

shock conditions (5) as the initial conditions. With the help of the solution so obtained and Simpson's formula, the energy efficiency function e_R is evaluated as

$$e_R = \frac{1}{\beta}\,\frac{I_2^2}{I_3}$$

where

$$I_2 = \int_0^1 f(h^2+\gamma g/f)^{1/2}\eta^2 \mathrm{d}\eta$$

$$I_3 = \int_0^1 (fh^2+\gamma g)\eta^2 \mathrm{d}\eta$$

The functions e_R is tabulated in Table 1 for different values of α and γ. For larger γ, the energy efficiency (for $\alpha=0$) decreases, as can be seen from Table 1. This fact is difficult to notice in the graphic representation of e_R in Ref. 1.

It is interesting to note that with increasing α, the energy efficiency decreases for lower γ while it increases for larger γ. Also for $0 \le \alpha \le 0.001$, the functional value of I_2 is affected by about 6-10%. However, for $\alpha > 0.001$, certain inconsistencies are observed during computation. Finally, the time t_s^* at which a point source again allows a finite mass flux is further reduced for increasing α and for all values of the coefficient of specific heat.

Acknowledgment

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Technical Comments

Comment on "High-Frequency Subsonic Flow Past a Pulsating Thin Airfoil"

Bhimsen K. Shivamoggi*
University of Colorado, Boulder, Colo.

PLOTKIN[1] developed high-frequency approximation for subsonic potential flow past a nonlifting pulsating airfoil. The method of solution given by Plotkin[1] requires a great deal of labor and is given as a Green's function source distribution along the chordline of the airfoil involving an asymptotic evaluation of the source integral using the method of stationary phase for the perturbation velocity potential. The purpose of this Note is to show that the problem in question lends itself so readily to the Laplace transform method that the development of the solution becomes almost trivial.

One has, for the linearized two-dimensional unsteady subsonic potential flow of an otherwise uniform stream with speed U, Mach number M, and speed of sound a past a thin nonlifting pulsating airfoil of chord l and thickness-to-chord ratio of $O(\epsilon)$,

$$(1-M^2)\phi_{xx} + \phi_{yy} - (2Mi\omega/a)\phi_x + (\omega^2/a^2)\phi = 0 \quad (1)$$

where the velocity potential is given by

$$\Phi(x,y,t) = Ux + \epsilon\phi(x,y)e^{i\omega t} \quad (2)$$

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*Department of Aerospace Engineering Sciences.

and the x axis is directed along the freestream and the chordline and the origin is at the leading edge.

If the surface of the pulsating airfoil is given by

$$f(x,t) = g(x)e^{i\omega t} \quad (3)$$

one has the linearized body boundary condition in the high-frequency approximation

$$\phi_y(x,0^{\pm}) = \pm i\omega g(x) \quad (4)$$

Upon taking the Laplace transform of Eqs. (1) and (4), as defined by

$$\tilde{\phi}(s,y) = \int_0^{\infty} e^{-sx}\phi(x,y)\mathrm{d}x \quad (5)$$

one obtains

$$\tilde{\phi}_{yy} + \mu^2\tilde{\phi} = 0 \quad (6a)$$

$$\tilde{\phi}_y(s,0^{\pm}) = \pm i\omega\tilde{g}(s) \quad (6b)$$

where

$$\mu^2 = [(1-M^2)s^2 - (2Mi\omega/a)s + (\omega^2/a^2)]$$

and, in the high-frequency approximation,

$$\mu \approx (\omega/a) - iMs$$

From Eq. (6), one obtains

$$\tilde{\phi}(s,y) = \frac{\omega\tilde{g}(s)e^{-i\mu|y|}}{\mu} \quad (7)$$

and, in the high-frequency approximation,

$$\tilde{\phi}(s,y) \approx -ae^{-i\omega|y|/a}\tilde{g}(s)e^{-M|y|s} \quad (8)$$